

To Cubeland and Back Again

Mathematical Aspects in the Minimal and Concept Art of the 1960s and 1970s

Mel Bochner – Donald Judd – Sol LeWitt – Ruth Vollmer

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In the 1960s, in a dense art scene in New York City, Minimal Art and Concept Art were born. The artists Carl Andre, Mel Bochner, Dan Flavin, Dan Graham, Donald Judd, Sol LeWitt and Robert Morris are subsumed into these two schools of art by contemporary critique and historical retrospection or simply credited as the protagonists of these movements. Additionally, the German immigrants, Hanne Darboven, Eva Hesse and Ruth Vollmer – the important links between the European and American avant-garde – could be found in this setting.¹

Many artists of this “New York Circle” were and are intellectuals with a distinct educational background, mostly with academic training, and worked as critics or docents, thereby shaping the discourse of the very art that they were creating.

The exemplary representation of the reception of mathematical methods and images by Mel Bochner, Donald Judd, Sol LeWitt and Ruth Vollmer show the relevance of mathematics within the “New York Circle” and the parallels with the intellectual history of “exact science.”²

The consideration of the relationship between the fine arts and mathematics in the 1960s, a decade of upheaval and new beginnings, is embedded in the prevailing *situation* of the relationship between art and science of that decade. This relationship had already begun to change for a certain period of time in America following the spirit of the age, in an atmosphere of increasing mathematization of science and society and emerging interconnections between art and science.

The Conquest of Space I – Space Travel and Consequences

¹ For this loosely connected group of artists, who were known to each other or were friends and exchanged ideas with one another, I will call in this text the “New York Circle.” Moreover many of these artists vehemently refused all their lives to be subsumed under one of the currents; the diversity of their works demonstrate this and do not build any single “movement”, “school” or “style”, as described by Donald Judd. See Donald Judd, “Specific Objects,” in: Arts Yearbook 8, 1965, p.74-82, here an in the following quoted after James Meyer (Ed.), *Minimalism*, London, New York 2002, p.207-210, here p.207.

² The consideration of the artworks is clearly not exhausted in this present analysis; a general survey of Minimal Art can be found in the following: James Meyer (Ed.), *Minimalism*, London, New York 2002.

*Okay, Houston, we've had a problem here.*³

John Swigert, 1970

While the hippies were celebrating at Woodstock during August 1969 with a wild music festival directed against the dominant and fossilized thought patterns of a regimented and conservative society and while they were expanding the narrowness of the socially accepted free space and their own consciousness, another group of Americans – technicians, engineers and scientists – were recovering from the festivities following the realization of the first manned moon mission in July 1969, a technological break-out from the human living space and with it the final conquest of space.

This moment was the culmination of a years-long, ideologically charged race in space travel, in which the tide had just turned. The Soviet Union had been the first nation to send an artificial satellite, Sputnik I, into space, on October 4, 1957, and the United States, completely astonished, realized the advantage of the Russians in space travel and suffered from “Sputnik-Shock.”

The events were superimposed on the “Two Cultures” debate, which had been unleashed by the English physicist and writer Charles Percy Snow in his lecture, “The Two Cultures and the Scientific Revolution” in 1959. His diagnosis of the relationship between the humanities and the natural sciences became a metaphor for the relationship between art and science, and shifted to the question of whether the USSR or the USA had the better scientific and educational system, which he decided in favor of the Soviet Union.⁴ Consequently, the American educational system was reorganized, and a change in the relationship between the arts and sciences took effect. New research centers and “creative centers” were established, such as the Center for Visual Studies at MIT or Harvard’s Carpenter Center for Visual Art, which were supposed to expand the connection between art and science. Additionally, art and technology movements, such as “Nine Evenings: Theatre and Engineering”, initiated by Billy Klüver and Robert Rauschenberg in October 1966, or E.A.T (Experiments in Art and Technology), also founded by them for the purpose of encouraging dialogue between artists and scientists, began to form. The program A&T

³ In the context of the educational system, which was recognized as deficient, the aerogram of John Swigert that was essentially wrongly quoted takes on a new meaning. The transliterated radio traffic of the Apollo-13 mission at the time of the accident can be found in the NASA Archive under <http://history.nasa.gov/Timeline/apollo13chron.html>.

⁴ Nearly exactly one year before the launch of the Sputnik-1 satellite on October 6th, 1956, C.P. Snow published the article “The Two Cultures” in *New Statesman*, which anticipated the “Rede-Lecture” (Snow, p.9). His lecture, which he held on May 7th, 1959 in Senate House in Cambridge as the “Rede-Lecture”, first enforced the idea and initiated a discussion lasting many years about the relationship of the humanities and the natural sciences: the so-called “Two Cultures” debate (Snow, p.57ff). The importance of Snow’s lecture regarding the development of the “cultures”, their relationship and their increasing approximation, is brought out again and again. See Helmut Kreuzer, *Die zwei Kulturen. Literarische und naturwissenschaftliche Intelligenz. C.P. Snows These in der Diskussion*, München 1987, or Nathalie Sinclair, David Pimm, William Higginson (Ed.), *Mathematics and the Aesthetic*, New York 2006, p.2.

(“Art and Technology”), beginning in 1967, saw the participation of 36 artists, among whom were Max Bill, Dan Flavin and Robert Rauschenberg.⁵ New scientific disciplines, like cybernetics, forced the mathematic saturation of the sciences, and at the same time computerization and digitalization proceeded and reinforced the growing reliance on mathematics in every day life.⁶

Donald Judd’s and Ruth Vollmer’s employment of newly developed materials in their works and their fabrication with industrial production techniques, Ruth Vollmer’s and Sol LeWitt’s contact with mathematicians, and Mel Bochner’s dialogues with scientists and the financing of his projects by E.A.T. all attest to the climate of open-mindedness concerning scientific and technological developments in which the artists of the “New York Circle” operated.

Numbers

...I used the Fibonacci series.⁷

Donald Judd, 1971

Donald Judd explained in a talk given in 1993, which can be considered a retrospective self-description of his oeuvre and mode of working: “Geometry and mathematics are human inventions. I use a small, simple proportion in my work for my purposes.”⁸ This betrays three things: Judd wants geometry to be understood as separate from mathematics – as was the case in the traditional canon of the seven liberal arts, where arithmetic and geometry were separate fields – and with this distinction, contemporary

⁵ It is certainly inadmissible to claim single events as causes and in this way to tell a teleological history, but both events, regarded from today’s point of view, are relevant and remain consequential for the relationship of the arts and the sciences. First of all, these two events, according to Christoph Klütsch, generated the environment in which early computergraphics could arise. See Christoph Klütsch, *Computergrafik. Ästhetische Experimente zwischen zwei Kulturen. Die Anfänge der Computerkunst in den 1960er Jahren*, Wien, New York 2007, p.37ff. and p.43f. About E.A.T and its activities see Douglas Davis, *Vom Experiment zur Idee. Die Kunst des 20. Jahrhunderts im Zeichen von Wissenschaft und Technologie*, Köln 1975, p.160ff.

⁶ Cybernetics is defined by Karl Steinbuch as a “summarization of several areas of science between technology and biology” (“Zusammenfassung mehrerer Wissenschaftsgebiete zwischen Technik und Biologie”), “which deals with control and feedback control procedures, in technology, in organisms and in communities” (“die Steuerungs- und Regelungsvorgänge behandeln, und zwar in Technik, bei Organismen und in Gemeinschaften”). He deems the methodology of cybernetics “as the intrusion of mathematical tools/methods in areas of sciences, in which until now seemed not to be practicable” (“als das Eindringen mathematischer Werkzeuge in Wissenschaftsgebiete, in denen sie bisher als nicht praktikabel erschienen”). With regard to the growing role of mathematics in many sciences (physics, logic, technics), which has already taken place, Steinbuch also predicts the saturation of the physiological, psychological and sociological disciplines in the 20th century. See Karl Steinbuch, *Automat und Mensch. Kybernetische Tatsachen und Hypothesen*, 2. extended edition, Berlin 1963, p.316ff.

⁷ See John Coplans, “Don Judd. An Interview with John Coplans”, in: *Artforum*, Vol.9, No.10 (1971), p.40-50, here p.47ff.

⁸ See Donald Judd, “Some Aspects of Color in General and Red and Black in Particular” (1993), in: Nicholas Serota (Ed.), *Donald Judd*, London 2004, p.145-159, here p.156.

sculpture also crumbles into two categories. Roughly speaking, it splits into a group in which geometrical forms are used, and another in which numbers are employed.⁹ Second, he reveals to his audience that he uses simple mathematical methods in his work. Third, he additionally shows his view concerning the ontology of mathematics – he speaks of “human inventions” – thus taking a constructivist position on mathematical modernism, in which the objects of mathematics are understood as human constructions, thereby standing in contrast to early positions in the philosophy of mathematics, such as Platonism.

What remains open in Judd’s retrospection is the purpose of the use of the mathematical methods; he does not divulge that here, but rather in elsewhere: “You see, the thing about my work is that it is given. Just as you take a stack or row of boxes, it’s a row. Everybody knows about rows, so it’s given in advance. Now, it’s also given if it’s a fairly simple progression, because everybody knows right off the spaces are given by mathematics. In one of the progressions I used the Fibonacci series. In another I used the kind of inverse natural number series: one, minus a half, plus a third, a fourth, a fifth, etc. No one other than a mathematician is going to know what the series really is. You don’t walk up to it and understand how it is working, but I think you do understand that there is a scheme there, and that it doesn’t look as if it is just done part by part visually. So it’s not conceived part by part, it’s done in one shot.”¹⁰

Series and relationships of numbers in arithmetic, which are also known as “arithmetic” or “geometric rows,” are employed for their own account in his works, such as in “Progressions”: for the creation of distances, of subdivisions within a form, or the arrangement of several similar forms within multi-part works, as well as for the creation of volumes, where the numerical proportions are especially able to serve in the creation of asymmetric proportioning. They are not the ideal, universally applicable proportions, like those used in antiquity or the Renaissance for anthropomorphic sculpture or in architecture, but instead Judd uses varied “schemes” in the formal composition of his work, which allows him to realize his “specific objects” and thus the linked concepts.¹¹

In 1965, Judd published the article “Specific Objects” as a remittance work in *Arts Yearbook*, in which he presented a critique of contemporary art and expressed his attitudes towards his own work. He postulated in his rejection of European art a new, multi-genre category of art, thought of as a holistic object without formal hierarchies, without the subordination of parts in favor of the unity of form: “The whole’s it. The big problem is to maintain the sense of the whole thing.”¹²

⁹ Interestingly, Nicolas und Elena Calas also distinguish contemporary sculpture in such a way. See Nicolas Calas, Elena Calas, *Icons and Images of the sixties*, New York 1971, p.271.

¹⁰ See John Coplans, “Don Judd. An Interview with John Coplans”, *l.c.*, p.47ff.

¹¹ On aesthetics of proportion see, for example, Umberto Eco, *Kunst und Schönheit im Mittelalter*, München 1993, p.49ff.

¹² See Bruce Glaser, “Questions to Stella and Judd”, in: *Art News*, Vol. 65, No.5, September 1966, quoted after Meyer, *Minimalism*, London, New York 2002, p.197-201, here p.198. Similar to his explanations in his famous

The mathematical rows allow the organization of the elements of the work in combination with a modified mode of production. The artwork does not emerge successively from the interplay of partial production, perception and reflection – da capo- but instead as the realization of a previously conceived idea. The “part by part” composition of the European tradition, which focuses on balance and harmony and which Judd rejects as anachronistic, does not bring about the art, rather it emerges in “one shot” and should thus fulfill the desideratum of “non-relational” art.¹³

Numbers serve in this case as production factors and are not metaphysical symbols. The reception of the Fibonacci numbers (1,1,2,3,5,8,13...), which Judd explicitly addresses, is not intended to create a metaphor for growth and the living, as is found with Mario Merz, who refers to Leonardo da Pisa’s (Fibonacci) population model developed in the early 13th century when he cites the Fibonacci series.¹⁴

On the contrary, the mathematical number series loses any original meaning for Judd when he brings these into the system of art, and he does not want his work to be understood as an illustration of mathematical topics: “The point is that the series doesn’t mean anything to me as mathematics, nor does it have anything to do with the nature of the world.”¹⁵ The meaning, explains Judd, as it is could only be understood by a mathematician in the language of mathematics. American artist Robert Smithson, a contemporary of Judd, speaks in this context of a “synthetic mathematics,” thus mathematics or geometry “that are separated from [their] original meaning,” because this mathematics is now only approached through the language of art and no longer through its original context.¹⁶

The Conquest of Space II

*...a definite space [passes] through a general space*¹⁷

Donald Judd, 1993

interview “Questions to Stella and Judd” with Bruce Glaser in 1964, Judd talks about his artistic challenge in his article “Specific Objects”: “The thing as a whole, it’s quality as a whole, is what is interesting.” See Judd, “Specific Objects”, quoted after Meyer, *Minimalism*, l.c., p.210.

¹³ See Coplans, “Don Judd. An Interview with John Coplans” l.c., p.47.

¹⁴ The description “Fibonacci-numbers” goes back to the Italian mathematician Leonardo da Pisa (Fibonacci) (circa 1170-1240). The numbers, appearing in his *Liber abaci*, had already been known in the context of a population problem of rabbits.

¹⁵ See Coplans “Don Judd. An Interview with John Coplans”, l.c., p.49.

¹⁶ See Robert Smithson, “Entropy and the New Monuments”, in: Jack Flam (Ed.), *Robert Smithson. The Collected Writings*, Berkeley, Los Angeles, London 1996, p.23. Here quoted after Davis Raskin, “Judd’s Moral Art”, in: Nicholas Serota (Ed.), *Donald Judd*, London 2004, p.79-95, here p.88.

¹⁷ This quotation refers to a description of the work *Untitled 1962* by Donald Judd himself: “The shell of this narrow space passes through the breath of the inner angle, a definite space through a general space.” See Judd, “Some Aspects of Color in General and Red and Black in Particular” (1993), l.c., p.148.

The conquest of space for the artists of Minimal Art was a necessary consequence, since the analysis of both art forms practiced to date – sculpture and painting, and especially painting – led to a critique of illusionism (inherent in these works). The necessary simulation of space and the role of portrayal (“Abbildfunktion”) in traditional painting were seen as anachronistic. The rejection of this led to an adjuration of an entity seemingly void of reference, to a new category of art and to the postulation that these deficits could be overcome by a departure from the picture and the use of space. Since “three dimensions are real space. That gets rid of the problem of illusionism and of literal space...” and this “actual space is intrinsically more powerful and specific than paint on a flat surface.”¹⁸

Donald Judd (1928-1994), after his military service as a surveyor at a Korean airport, settled down in 1948 in New York and began his studies of Fine Arts at the Arts Students League, and shortly thereafter started studying art history and philosophy at Columbia University. As an artist, he first emerged as a painter, but in the early 1960s he began to produce sculptures and plastic works instead of the abstract-expressionistic painting that he, like many of his colleagues, had practiced before.¹⁹ Like many artists of the “New York Circle,” he also became known as a critic, writing in several art magazines.

In 1962, his first freestanding work, *Untitled 1962*, appeared. This piece deals with open and enclosed space. In an arched and closed pipe, which connects two wooden plates, both painted red and standing at right angles, “linear space” runs through the surrounding space, or, as the artist himself explains, “narrow space passes through the breadth of the inner angle, a definite space through a general space.”²⁰ Judd describes programmatically the newly gained liberty and the studies done systematically in the ensuing years: “Obviously, anything in three dimensions can be any shape, regular or irregular, and can have any relation to the wall, floor, ceiling, room, rooms or exterior or none at all.”²¹

It seems as if Sol LeWitt were also following this program, since the first *Structures*, as he called his room-filling pieces, emerged at this time (*Wall Structure Blue*, 1962). Like Judd, he also probed the relationship between artwork and its surrounding space, with pieces standing on the floor (*Floor*

¹⁸ Judd discusses the different illusions of space, which are unavoidable in traditional painting. Even in a non-figurative painting with two colors, a form of a relief occurs, Judd tells us, because one of the colors presses to the foreground and the other one to the background, while the usage of only one color creates a sense of infinity. Therefore, in all cases there is an illusion of space. See Judd, “Specific Objects”, l.c., p.209.

¹⁹ For Judd, the space represents, along with material and light, one of the three basic conditions of the fine arts. See Judd, “Some Aspects of Color in General and Red and Black in Particular” (1993), l.c., p.145. Space is of almost existential importance to him, as he once described in a retrospective. See Donald Judd, “Kunst und Architektur”, in: Gregor Stemmerich (Ed.), *Minimal Art. Eine kritische Retrospektive*, Basel, Dresden 1995, p.74-91, here p.85f.

²⁰ See Judd, “Some Aspects of Color in General and Red and Black in Particular” (1993), l.c., p.148.

²¹ See Judd, “Specific Objects”, l.c., p.209-210.

Structures), affixed to the wall (*Wall Structures*) or hanging from the ceiling (*Hanging Structures*), and similarly played with the relationship between inner and exterior space, with interpenetrating forms.²²

Solomon LeWitt (1928-2007), like Donald Judd, is considered to be one of the protagonists of Minimal Art as well as their bridge to Concept Art. He moved to New York in 1953; after completing his studies in Fine Arts at Syracuse University, he worked as a graphic designer for various magazines and as a draughtsman for architect I.M. Pei. Through his contact with the artists Dan Flavin, Robert Mangold, Robert Ryman and the critic Lucy Lippard, who he met through his job at MoMA (Museum of Modern Art), he received the important impulses for his artwork of the early 1960s, which can be considered his breakthrough as an artist. These are for the most part composed of simple, three-dimensional, often rectangular geometric forms, which started his journey, like so many of his contemporaries, to “Cubeland.”²³ The cube especially, which serves as a basic grammatical element in his artistic language, became an essential modular “basic unit” in his oeuvre.²⁴ In the European tradition, the cube as a Platonic solid still stands for harmony, perfection and beauty (Rune Mields) – the polyhedron served as a building block for Plato’s cosmological model in his *Timaios* Dialogue.²⁵ LeWitt, however, used the properties of this timeless, stereometric basic form, which, in contrast to everyday objects, are less associative, to avoid expression and subjectivity. He retrieves the cube from metaphysical space and brings it down to the weighty ground of facts, as he humorously states in his paper “The Cube”, “The most interesting characteristic of the cube is that it is relatively uninteresting.”²⁶ While Judd renewed the appearance of the geometric forms through the use of new materials and finishes, LeWitt freed them from historical implications by integrating them into structural systems. The use of simple geometric base forms yields a repertoire of pure forms, whose symbolic and referential charge is largely neutralized.

The edges of his *Structures* are initially unfinished and show the natural color of the material. Soon, however, they become a monochrome black, before being painted white. This allows the structure

²² Sabeth Buchmann shows through a comparison the impulses that LeWitt received from the artwork of Russian Constructivist Alexander Rodtschenko. See Sabeth Buchmann, *Denken gegen das Denken, Produktion, Technologie, Subjektivität bei Sol LeWitt, Yvonne Rainer und Hélio Oiticica*, Berlin 2007, p.149ff.

²³ “Cubeland” according to “Spaceland” is the world of three dimensions in Edwin A. Abbotts didactic romance *Flatland. A Romance of Many Dimensions*. In contrast to the two-dimensional “Flatland,” “Spaceland” is located with a cube on the map of the lands of different dimensions. Additionally, the artists of Minimal Art were identified – often derisively – with the cube or box. LeWitt must have known this book, as can be deduced through a close observation of one of his artworks: a photo series titled *autobiography*, in which he systematically photographed his studio in New York, in a typical style using a uniform, square format, which suggests a unification and an objectifying of the view. Some of these photographs are exposures of his private library and provide an insight into his intellectual world, in which the novel of the English teacher Edwin A. Abbott (alias A. Square) can also be found. See A. Square, *Flatland. A Romance of Many Dimensions*, London 1884 and George Stolz (Ed.), *Sol LeWitt. Fotografía*, Madrid 2003.

²⁴ See Sol LeWitt, “The Cube”, in: *Art in America*, 1966

²⁵ See Platon, *Werke in Acht Bänden, Band 7, Timaios – Kritias – Philebos*, Gunther Eigler (Ed.), 2. unchanged edition, Darmstadt 1990, 53c-55c.

²⁶ See LeWitt, “The Cube”, l.c.

to come more boldly to the fore. This is where he leaves Judd's path, as he is not interested in the sensual qualities of the material, but instead in reducing them in favor of the system.²⁷

Beginning in 1963, the artist Ruth Vollmer (1903-1982) was engaged with another "minimal" basic geometric form: the sphere. She investigated the content of this figure in an intensive, artistic research project, while at the same time looking into the mathematical properties of space: "I am involved with the sphere; exploring it geometrically, and finding unexpected forms."²⁸

Ruth Vollmer arrived as a Jewish immigrant to New York in 1935 and was of an older generation than most of the artists in the "New York Circle."²⁹ Both her different cultural background – Vollmer, as a European and a contemporary, acted as a bridge to the European pre-war avant-garde, to Bauhaus and Russian Constructivism – and her mathematical knowledge made her person and her work interesting for some artists in the "New York Circle."³⁰ She developed close friendships especially with Eva Hesse and Sol LeWitt, but Robert Smithson and Mel Bochner were also guests in her informal "Salons."

She was excited by the geometric form and its properties and produced an entire series of bronze sculptures. Especially the unpolished bronzes had at the time of "dematerialization" in art haptic surfaces and were therefore reminiscent of antiquity, which Vollmer had loved, and of the early discoveries of Platonic solids, which can again all be inscribed within the sphere.³¹

At the same time, Barbara Hepworth (1903-1975) and Max Bill (1908-1993), both of the same generation as Vollmer, grappled with the "ball" and its shell. Georges Vantongerloo (1886-1965), a friend of Bill, produced in the last year of his life his sculpture *Une étoile gazeuse*, a sphere made of plexiglass. Hepworth's *Sphere with Inside and Outside Colour* (1967) and Bill's *Familie von 5 halben Kugeln* (1965/1966) are exemplary in the deepening exploration of this theme.

²⁷ Unlike Judd, LeWitt gives his works self-explanatory titles, which later, in his conceptual works, become even more important because with them he can shift the focus onto the underlying concept and its representation as text.

²⁸ See Nadja Rottner, "Thinking the Line", in: Nadja Rottner, Peter Weibel (Ed.), *Ruth Vollmer 1961-1978, Thinking the Line*, Stuttgart 2006, p.61 and p.203.

²⁹ Ruth Vollmer (born Landshoff) grew up in an artistic home in Munich. Her mother Phillipine Landshoff was a singer, her father Ludwig Landshoff was a musicologist and conductor, and her paternal uncle was the publisher Samuel Fischer. The houses of her parents and her uncle were attended by many musicians, writers, artists and scientists, such as Thomas Mann and Albert Einstein; this was the world in which she lived. To continue with this tradition, her apartment in Manhattan became a "Salon" for many artists of the "New York Circle." For biographical information on Vollmer see Nadja Rottner, Peter Weibel (Ed.), *Ruth Vollmer 1961-1978, Thinking the Line*, Stuttgart 2006, p.116ff.

³⁰ Vollmer exchanged letters with Naum Gabo because she was attracted to his abilities and interests. Gabo was able to manufacture mathematical models because of his academic education as an engineer. See Anna Vallye, "The Reenchantment of the World: Ruth Vollmer's Science", in: Nadja Rottner, Peter Weibel (Ed.), *Ruth Vollmer 1961-1978, Thinking the Line*, Stuttgart 2006, p.98-115, here p.100.

³¹ These works were conceived to stand on the floor – works of Minimal Art were generally presented without a base or pedestal – and were shown in the exhibition *Ruth Vollmer: Sculptures, Spheres* in 1966 in the Betty-Parson Gallery.

In studies that are almost didactic, Vollmer shows her interest in extremes such as the concave/convex or the complete/incomplete as well as in intersections, when for example cubic forms intersect spherical ones and the resulting cross-section is removed – as seen in *Sphere with Square Incision* (1963) or *Monomer* (1965). Over and over, multiple, interpenetrating spheres are joined together, placed inside one another or juxtaposed with complementary forms. Above all, her attention is captured by the strange case in which the arched surface, when viewed from outside, suggests an interior space that is actually completely different. These are “introductions to mathematical ideas” of space, and they foreshadow her later work in which she addresses more complex mathematical figures – since non-Euclidian geometry is already present on the surface of the sphere.³²

Another approach to space can be seen in the work of Carl Andre (born 1935), *The Void Enclosed by Three Squares of Three, Four and Five* (1997), in which he uses equal sized and very flat metal plates to build three squares that touch to define an enclosed space, which upon closer examination can be identified as a right triangle.

Andre wants the space above each of the plates to be regarded as “column[s] of air;” today one could speak of virtual space. In such a “flat” plastic work, which he calls “clastic”, because it is comprised of single pieces without connection and is loosely assembled, the question of the boundary between two and three-dimensionality is raised.³³ Particularly in the presented work, in which the enclosed space is not completely bordered by the surrounding plates in every spot, in contrast to his work *Cuts*, he also questions the concept of single dimensionality. The tapering sides of the triangle suggest: there are three points – the vertices of the triangle – which in contrast to all others, are not defined from the outside. Euclid explained in his *Elements*, “a point is that which has no part.”³⁴

The Pythagorean theorem inspired the structure of the piece, and the work can also be regarded as an homage to the music that Andre was very attached to (Bach), since the school of Pythagoras developed an early music theory based completely on integers and built a connection between proportions and music.³⁵ In the piece *20 Cedar Slant 20°*, he placed 20 rectangular cedar blocks next to each other at an angle of 20 degrees to the wall, thereby building a special constellation of space.

³² “Introduction to Mathematical Ideas” is the title of a mathematical book whose elegantly reduced cover shows the two-piece sculpture *Complementary Forms* (1967) of Ruth Vollmer, thus further underscoring the interconnection between arts and mathematics at this time. See David G. Crowdis, Brand W. Wheeler, *Introduction to Mathematical Ideas*, New York 1969.

³³ See Phyllis Tuchman, “An Interview with Carl Andre”, in: *Artforum*, Vol. VIII, No.10, June 1970, p.55-61, here p.57 and p.60-61.

³⁴ Euclid gave the definition: “Punctum est, cuius pars nulla est.” See Euklid, *Die Elemente*, Clemens Thaer (Ed.), Darmstadt 1991.

³⁵ See Eva Meyer-Hermann (Ed.), *Carl Andre, Sculptor 1996. Krefeld at home. Wolfsburg at large*, Stuttgart 1996, p.23.

While Ruth Vollmer addressed mathematical models of non-Euclidian geometry and Sol LeWitt structured space, Judd experimented with the configurations of object/wall and object/corners until he finally related single objects to the otherwise entirely empty surrounding space, thus bringing the relationship of object to environment to the fore.³⁶ In this way, the object-space and the surrounding space are connected not only through direct contact (without pedestals), but also through the meshing of interior and exterior space – for example in *Stacks* – and through the mirroring of the surrounding space in the very reflective surface of the piece, so that the object and the environment merge – as in the exhibited work *Untitled 1985*. He had already identified the environment in his description of his work *Untitled 1985* as “general space” and positioned it against “definite space.”³⁷ This general space, an empty and actually existing room, is a Euclidian space in which right angles dominate and in which Judd defines subspaces.

Although Judd’s statements in interviews and discussions continually reveal him to be an artist who engages in scientific, specifically mathematical, theories – “the only stance that the contemporary artist should take towards science is that the appearance and implications of art should correspond to what is known today”³⁸ – the sculptor Mark Di Suervo accuses him in a panel discussion during the exhibit *Primary Structures* at the Jewish Museum in New York of ignoring the mathematical conceptions of space when discussing it. He further asserts that the space we perceive is not true space and that it should rather be thought of as “warped.”³⁹

With this statement, Di Suervo refers to the warped spaces and their geometries that were developed in the 19th century in differential geometry and Riemannian geometry and were applied to physics at the start of the 20th century.⁴⁰

Judd advocates the separation of the individual discourses within mathematics and the fine arts, and refuses an art that would address such geometric issues when he says: “I think there’s a big gap

³⁶ These cubic or block-shaped objects often appear as repeated elements arranged in rows in his *Progressions* or *Stacks*, where they create a uniform arrangement. Judd created his first wall sculpture in 1965, a so-called *Stack*, which grows from the floor to the ceiling. See Judd, “Some Aspects of Color in General and Red and Black in Particular” (1993), l.c., p.149.

³⁷ See Judd, “Some Aspects of Color in General and Red and Black in Particular” (1993), l.c., p.148.

³⁸ See Donald Judd, “Art and Architecture”, in Donald Judd, *Complete Writings 1975-1986*, Eindhoven 1987, p.25-36.

³⁹ See Mark di Suvero, Donald Judd, Kynaston McShine, Robert Morris, Barbara Rose, “Symposium on *The New Sculpture*”, New York, 2. Mai 1966, in: James Meyer, *Minimalism*, London, New York 2002, p.220-222, here p.221.

⁴⁰ See Klaus Mainzer, *Geschichte der Geometrie*, Mannheim, Wien, Zürich 1980, p.134ff.

between the discussion of Euclidian and non-Euclidian geometry and art, and that the two should be left in the different areas in which they are.”⁴¹

This is amazing that Judd could apply these terms of warped spaces for his works and that he had already coined neologisms. Since the development of an “inner geometry” within differential geometry and later in Riemannian geometry, geometric objects can be thought of as without the surrounding space in which they are embedded.

Yet Judd instead celebrates the concept of the Euclidian space, when he builds and presents almost rectangular and brick-shaped, or right-angled, objects. It appears as if he wants his “specific objects” to be perceived within a Euclidian space, thus accessible to human senses and outlook, and not those of the imagined or speculative space. Thus he reinforces his position as an empiricist and makes his objects even more “real.”⁴² To the same extent that modern artists like the Suprematists or Surrealists filled the “free space” of non-concrete, higher dimensional and non-Euclidian space with their own interpretations, so Judd uses Euclidian space to construct his stark and solemn artworks in the moment of perception.⁴³

(Primary) Structures

*And then I had a mathematician check it out.
Somebody did find a mistake I had made.
And I was very happy! (laughing)*⁴⁴
Sol LeWitt, 1979

Like an architectural building, the partially opened surfaces of Sol LeWitt’s early *Structures* allow a view into the interior of the coated space. With the increasing stripping of these structures until the mid 1960s, they became “naked,” showing their bones – the underlying grid. Carl Andre describes this process of

⁴¹ Because Judd uses the terms “Euclidian” and “non-Euclidian geometry” independently, reacting to the keyword “warped space” given by Di Suvero, it is evident that he knows the discourse. See Mark Di Suvero, Donald Judd, Kynaston McShine, Robert Morris, Barbara Rose, “Symposium on *The New Sculpture*”, l.c., p.221-222.

⁴² One tends to conclude this when momentarily ignoring the arguments of Hans Hahn, who showed in his article “Die Krise der Anschauung” (1933), that all geometries, and therefore also Euclidian geometry, are constructions. Nevertheless Euclidian space is still the space that is the closest to our daily experiences. See Hans Hahn, “Die Krise der Anschauung”, in: Brian McGuinness (Ed.), *Empirismus, Logik, Mathematik*, Frankfurt/M. 1988, p.86-114.

⁴³ About the reception of non-Euclidian geometry in the fine arts of the 20th century before the 1960s see Linda Dalrymple-Henderson, *The fourth Dimension and Non-Euclidean Geometry in Modern Art*, Princeton 1993.

⁴⁴ Sol LeWitt in a video interview with Russell Bowman (Director of the Museum of Contemporary Art in Chicago [MCA]) in 1979, in which he describes the concept and the development of his work *Incomplete Open Cubes* (1974). Transliteration is by the author of this article. See Videotape “Interview with Sol LeWitt” – Archive of American Art and Museum of Modern Art Chicago.

structural exposure with two architectural examples: Bartholdi's Statue of Liberty in New York City and the Eiffel Tower in Paris.⁴⁵

The transition from form to structure in Sol LeWitt's work is an exemplary reflection of the development within the fine arts of the humanities and the natural sciences, especially mathematics in the 20th century. The discipline of mathematical Modernism had changed under the direction of David Hilbert into a science of structures, whose foundation was formal logic and the conception of the set, upon which abstract structures could be built. Among other things, this was a reaction to the "crisis of intuition" ("Krise der Anschauung"), which is how Hans Hahn describes the loss of confidence in images within mathematics due to the appearance of "monsters",⁴⁶ "non-Euclidian" and "higher-dimensional" geometries and how he also describes the split between language and images.⁴⁷

The modern mathematics of structure, as a self-referential language, no longer claims that a connection to an empirical or metaphysical reality should be a secure foundation for the practice of this discipline. The single objects were no longer observed, but instead the objects and their relationships to each other became the focal point and should provide "insights" to underlying coherences when seen in analogy with Sol LeWitt's *Structures*. The mathematical spaces also became structures, sets grafted with properties, and LeWitt's art thus shows in many ways surprising correspondences with the mathematical practices of the time.

Moreover, the science of structures was reactivated due to the reorganization of the American educational system starting in the late 1950s, as mentioned above, and implemented as "New Math" in the curricula of both schools and universities, thus being made public to a broad audience.⁴⁸ The structural mathematics became an approach of the "intellectual movement of structuralism."⁴⁹ Both played prominent roles in the 1960s and the relationships to the fine arts were already established.⁵⁰ As one of

⁴⁵ See Tuchmann, "An Interview with Carl Andre", l.c., p.55.

⁴⁶ A "monster" is a mathematical situation that serves as a limiting exception ("limitierende Ausnahme") or has an unexpected, paradoxical character („unerwarteten, ja paradoxen Charakter"). In a narrower sense, it is a mathematical object whose existence seems to be impossible due to intuitive reasons („mathematisches Objekt, dessen Existenz aus Gründen, die der Anschauung entnommen sind, unmöglich erscheint.“ See Klaus Volkert, *Die Krise der Anschauung. Eine Studie zu formalen und heuristischen Verfahren in der Mathematik seit 1850*, Göttingen 1986, p.101 and p.136.

⁴⁷ On the crisis of intuition see Hans Hahn, "Die Krise der Anschauung", l.c.

⁴⁸ See Herbert Mehrrens, *Moderne – Sprache – Mathematik. Eine Geschichte des Streits um die Grundlagen der Disziplin und des Subjekts formaler Systeme*, Frankfurt/M. 1990, p.320.

⁴⁹ See *ibid.*, p.7.

⁵⁰ Hans Naumann published the book *Der Moderne Strukturbegriff* in 1973 for given reasons, in which different authors trace the historical genesis of modern structural terminology. Following Naumann, the 1930s can be regarded as the threshold to the modern notion of structure, and since this time the concept found its way into many sciences, culminating in the movements of structuralism in the 1960s. See Hans Naumann (Ed.), *Der Moderne Strukturbegriff. Materialien zu seiner Entwicklung*, Darmstadt 1973, p.1ff. Gyorgy Kepes, as editor, published the German translation of his book *Struktur in Kunst und Wissenschaft*, in 1967 where the notion of structure became viable for fine arts, especially concrete art. See Gyorgy Kepes (Ed.), *Struktur in Kunst und Wissenschaft*, Brüssel

the fathers of Concrete Art, Bill published in 1965 his article “struktur als kunst? kunst als struktur?” in which he describes “structure” as a new way of creation through experimenting with “design configurations”, in order to include new aesthetic domains within the concept of art.⁵¹

With the introduction of the modular concept in his art (*Five Modules with One Cube*), LeWitt connects both single and multiple cubic base units to a quadratic framework, a grid, which can be used as a homogenous playing field (*Modular Cube/Bases*, 1968). The potential dynamic of the components, which can be moved along the playing field, anticipates his serial and multipart works (*Cubes with Hidden Cubes*, 1968).⁵²

Serial Project No.1 ABCD (1966), which can be considered one of his masterpieces, is accompanied by documentation and commentary. It shows within four sets, each consisting of nine subdivisions, the variations of nested cuboids resulting from the systematic change of three independent parameters (height, inside/outside, open/closed). The artist emphasizes over and over that the serial method is indebted to his strong interest in film sequences and the photographic series of Eadweard Muybridge. The conceptual quality becomes more important in comparison with the material and spatial properties because of the relationship of several parts with the structure of higher order structures – systems.⁵³

One can see that LeWitt, in contrast to Judd, is more interested in systemic variations and their visualizations, though sometimes he only offers partial solutions of his systems and leads the viewer to complete it in his or her mind’s eye. In this way, one must distinguish between artwork done in a series and serial artwork, as Mel Bochner explains in his article “The Serial Attitude”.⁵⁴

The comparison between Max Bill’s graphical portfolio *quinze variations sur un même thème* done as a series in the 1930s with the serial works done by LeWitt demonstrates that the grade of formalization defines the nuances. The Swiss artist, starting with one basic idea (“Grundidee”), presents

1967. Marcelin Pleynet tries to tie contemporary art to structuralism. See Marcelin Pleynet, “Malerei und Strukturalismus”, in Gregor Stemmerich (Ed.), *Minimal Art. Eine kritische Retrospektive*, Dresden, Basel 1995, p.418-443.

⁵¹ See Max Bill, “struktur als kunst? kunst als struktur?”, in: Gyorgy Kepes (Ed.), *Struktur in Kunst und Wissenschaft*, l.c., p.150. Earlier in Gyorgy Kepes (Ed.), *Structure in Art and Science*, New York 1965.

⁵² Sabeth Buchmann describes the grid structure („Gitterstruktur“) according to Dirk Snauwaert and Lucy Lippard as an “emblem of modernity” (“Emblem von Modernität”), which developed as a result of the second industrial revolution associated with standardization in mass production. See Buchmann, *Denken gegen das Denken, Produktion, Technologie, Subjektivität bei Sol LeWitt, Yvonne Rainer und Hélio Oiticicia*, l.c., p.149.

⁵³ According to LeWitt, space can be easily structured by using a base unit: “Space can be thought of as the cubic area occupied by a three-dimensional volume,” if the space is not so meaningful for the artwork: “If space is relatively unimportant it can be regularized and made equal.” See Sol LeWitt, “Paragraphs on Conceptual Art”, in: *Artforum*, Vol. 5, No. 10 (June 1967), p.79-83, here p.83.

⁵⁴ Mel Bochner, with Morandi’s “Bottles” and de Kooning’s “Woman,” typifies the artist who works in series, whereas serial order appears as a method and not as a style in the works of Duchamp, Judd and LeWitt. Moreover, Bochner describes modularity as a repetition of base units in serial arrangements. See Mel Bochner, “The Serial Attitude”, in: *Artforum*, Vol. 6, No. 4 (December 1967), p.28-33.

15 variations of the same theme within his graphic pages: all regular figures from the triangle to the octagon are placed within each other, with the base of the former shape becoming the new side of the next shape.

While Bill pleads for the potential of concrete art with its infinite possibilities through the multiple execution of one theme of departure by means of geometry (“Werkmaterial”), which already differs from LeWitt as representation of an idea through pictures rather than a system of rules, LeWitt follows a regular system to move away from subjective, emotional art to an objective, verifiable and transparent “experimental design” (“Versuchsanordnung”).⁵⁵

The rules of his closed systems are arbitrary and completely determine the appearance of the work according to the chosen formal means – “the idea becomes a machine.”⁵⁶ This occurs by arbitrarily setting the premises and the axioms of his deductive system and systematically deriving the possible configurations, or – as he describes it in his *Sentences on Conceptual Art* – “irrational thoughts should be followed absolutely and logically.”⁵⁷ The degrees of freedom are lower in LeWitt’s systems and he embodies the Homo ludens less than Bill, for whom it was a pure play of form and color (“reines spiel von form und farbe”)⁵⁸ and who kept himself open to the creative impulse.⁵⁹

Later, over and over LeWitt would breach and play with ironies in his systems through paradoxes or incomplete realizations, and he was delighted when a mathematician would check the validity of his system, such as with the mathematical language of *Incomplete Open Cubes* (1979): “and then I had a mathematician check it out. Somebody did find a mistake I had made. And I was very happy!

⁵⁵ Bill on his work: “Within these closely drawn boundaries, there are so many possibilities for variation that a single theme, a single basic idea, can lead to fifteen very different images, proving that the concrete art contains an infinite number of possibilities.” (“innerhalb dieser eng gezogenen grenzen liegen so viele variationsmöglichkeiten, dass man schon darin, dass ein einziges thema, das heisst eine einzige grundidee, zu fünfzehn sehr verschiedenen gebilden führt, einen beweis erblicken kann, dass die konkrete kunst unendlich viele möglichkeiten in sich birgt.”) See Max Bill, “fünfzehn variationen über ein thema”, in: Eduard Hüttinger, Max Bill, Zürich, Stuttgart 1987, p.80-81, here p.80. In addition to this, “objectivity” appears already in 1962 as the title of Sol LeWitt’s painting *Objectivity*.

⁵⁶ LeWitt formulates: “The idea becomes a machine that makes the art.” See LeWitt, “Paragraphs on Conceptual Art”, l.c., p.79.

⁵⁷ See Sol LeWitt, “Sentences on Conceptual Art”, in: *Art-Language*, Vol. 1, No.1 (May 1969), p.11-13.

⁵⁸ See Bill, “fünfzehn variationen über ein thema”, l.c., p.81.

⁵⁹ This becomes apparent when Bill talks explicitly about the temperament of the artist: “such legitimate constructions, which are developed outside of any schematic proportioning and entirely from their own inner reality, can be constructed completely differently according to personal desire and temperament for any freely chosen topic. They then lead to the most diverse images depending on the choice of topic – be it complicated or simple.” („solche gesetzmässige konstruktionen, welche ausserhalb jeder schematischen proportionierung, lediglich aus ihren inneren gegebenheiten entwickelt sind, können je nach persönlichem willen und temperament, auf jedem frei gewählten thema, vollkommen andersartig aufgebaut werden und je nach wahl des themas, ob kompliziert, ob einfach, zu den verschiedensten gebilden führen.“) See Bill, “fünfzehn variationen über ein thema”, l.c., p.80.

(laughing).”⁶⁰ The work *Form derived from a cube* shows in a playful way a few variations that emerge from the variations on a cube, and thus the breaking-through of formal strictness of his earlier years.

Beginning with his *Walldrawings*, LeWitt returned to the beginning of art and his own art, coming back to “Flatland”.⁶¹ These works also mark the crossover to Concept Art, since in the end the idea of the work is more important than the actual material manifestation of it. The artist delivers as an author only the description of the future work, while the actual production of the drawing is ceded to others, often employees. In his “Paragraphs on Conceptual Art”, LeWitt writes: “Conceptual art doesn’t really have much to do with mathematics,”⁶² which means, in other words, “something already,” and he immediately adds, “the mathematics used by most artists is simple arithmetic or simple number systems.”⁶³

The aesthetic art is coded in a language, which also becomes part of the artwork. He uses numbers for the coding of the *Wall Markings* (1968), in which certain hatchings are permuted and finally interfered into a grid.

For the description, alphanumeric sign systems are used, which belong no more to mathematics than to literature. According to LeWitt, “if words are used, and they proceed from ideas about art, then they are art and not literature, numbers are not mathematics.”⁶⁴

During the time in which the first exhibitions of computer graphics took place in New York City, a closeness to the digital picture developed, which is likewise represented by symbols. When LeWitt says, “the artist would select the basis form and rules that would govern the solution of a problem,”⁶⁵ he almost provides the current definition of an algorithm, thus his work also approaches an algorithm, which has a long mathematical tradition and can be regarded as an interface between art and mathematics.⁶⁶

The Pictures of Space and Beyond

⁶⁰ Like annotation 44.

⁶¹ His *Walldrawings* are also references to cave paintings and frescos. LeWitt started as a painter, worked with a flat medium and then wanted to use a medium that was maximally two-dimensional, as seen when he says: “I wanted to do a work of art that was as two-dimensional as possible.” See Sol LeWitt, “Wall Drawings”, in: Gary Garrels (Ed.), *Sol LeWitt. A Retrospective*, San Francisco 2000, p.375.

⁶² See LeWitt, “Paragraphs on Conceptual Art”, l.c., p.80.

⁶³ See *ibid.*

⁶⁴ See LeWitt, “Sentences on Conceptual Art”, l.c., p.11-13.

⁶⁵ See LeWitt, “Paragraphs on Conceptual Art”, l.c., p.80.

⁶⁶ For Jochen Ziegenbalg the algorithmic method (“Algorithmik”) is an important interface between the fine arts and mathematics. See Jochen Ziegenbalg, “Algorithmik als Schnittstelle zwischen Kunst und Mathematik”, in: Barbara Könchen, Peter Weibel (Ed.), *unsICHTBARes, Algorithmen als Schnittstellen zwischen Kunst und Wissenschaft*, Bern 2005, p.174-199.

*The mathematical form is so beautiful because it's so objective!*⁶⁷

Ruth Vollmer, 1975

Beginning in 1968, Ruth Vollmer turned to the image worlds of pre-modern mathematics. Mathematical models from the 19th century, such as “Pseudosphere” (1868) or “Steiner’sche Fläche”, which Vollmer discovered through tips at Columbia University, became the base of her sculptures *Pseudosphere* (1970) and *Six Intersecting Ovals* (1970).⁶⁸ The artist was fascinated by these figures because they were not abstractions of geometry – elementary, stereometric forms found by observing nature – but based on formulas. She excitedly explained, “on mathematical formulas, not on geometry! It is very interesting.”⁶⁹ It is about mathematics, which manifests itself concretely in models (“plastisch manifestiert”), and “from this comes an indisputable aesthetic effect,” (“es geht von diesen eine unbestreitbare ästhetische Wirkung aus”) as was argued by Max Bill in 1949 in his article, “die mathematische denkweise in der kunst unserer zeit” and points to the possibilities of the inclusion of new domains in art and the expansion of artistic modes of expression. These figures are not merely aesthetic forms, according to Bill, but they are “thoughts, ideas, insights that have become form.” (“form gewordener gedanke, idee, erkenntnis”)⁷⁰

Through the reception of these models, Vollmer returns to the common strategy of “visualization” in fine arts and mathematics, reflecting the long tradition in mathematics of producing visual models. She also deals with the long and shifting discourse on the role of visual media and its status in mathematics.⁷¹

Already in antiquity models of Platonic or Archimedic solids were manufactured, which can be regarded as early images of mathematics. Interestingly, the drawing of the icosahedron in Euclid’s *Elements* can be considered as the first image of a complex figure in the history of images in mathematics; Vollmer creates such an icosahedron out of wire, which is shown in one of her exhibition catalogues.⁷²

⁶⁷ See Susan Carol Larsen, “An Interview with Ruth Vollmer”, in: Nadja Rottner, Peter Weibel (Ed.), *Ruth Vollmer 1961-1978, Thinking the Line*, Stuttgart 2006, p.206-211, here p.210.

⁶⁸ Ruth Vollmer: “For years and years I have been interested in mathematical forms, in geometry, and for a long time in mathematics. And somebody told me Columbia [University, New York City] has mathematical forms.” See *ibid.*, p.210 Ruth Vollmer worked together with the physicist Erna Herrey, who was an adviser to her.

⁶⁹ See *ibid.*

⁷⁰ Bill also mentions that these forms, like structures of the world („strukturen des weltgefüges“), give a picture (not an image) of the world, as is known today. See Max Bill, “die mathematische denkweise in der kunst unserer zeit”, in: Eduard Hüttinger, *Max Bill*, Zürich, Stuttgart 1987, p.117-128, here p.126.

⁷¹ Like Nadja Rottner points out: “Ruth Vollmer’s sculptural works investigate the shared potential of art and mathematics.” And: “Vollmer touched upon a fundamental feature of mathematical thinking: the role and function of the model. A model is not a translation of the formula; it is the equivalent of the formula.” See Rottner, “Thinking the Line”, *l.c.*, p.59 and p.60.

⁷² Vollmer’s *Icosahedron*, an icosahedron made of wire circa 1965 was photographed by her brother Hermann Landshoff. See cover of the exhibition catalogue: Jil Dunbar (Ed.), *Ruth Vollmer 1903-1982*, New York 1983.

The construction of models in recent times began in the field of French geometrist Gaspard Monge at the *École Polytechnique* in Paris; the shift from drawing to three-dimensional models for teaching and studying higher geometry can be found here.⁷³ A “boom of images” came over mathematics in the mid-19th century as models of special surfaces of differential geometry and the first (abstract) models for non-Euclidian geometry were developed and were enthusiastically produced, above all, by Felix Klein – later also by publishing companies – on a larger scale.⁷⁴

The construction of such models is based, in general, on the visualization of a set of points that emerge from functions or solution sets of algebraic equations and are placed in a suitable and mostly spatial coordinate system.

This visualization method was made possible through the innovations of René Descartes and Pierre de Fermat in analytical geometry during the 17th century. Through the interweaving of geometry and algebra, the exchange of image and calculus was enabled: geometric problems could now be solved with the help of algebra and algebraic equations could be made concrete.⁷⁵

The visual media of mathematics was always historically relevant, considering the generation and formation of new innerdisciplinary knowledge and the influence on the development of mathematical language. One could say: an interplay of medial conditionality between visual media and the text of mathematical language.

The heuristic – leading to understanding – function of images in mathematics in these processes is recognized, for example, in revealing qualitative or global properties, whereas the function of foundational knowledge is controversial, for example the proof of existence through images, since in contrast to the strict language of mathematics, pictures are less precise and can be interpreted in multiple

Benno Artmann treats the drawing of the icosahedron in a surviving copy of Euclid's *Elements* (Manuscript P) as the first diagram of a complex geometric object. The icosahedron can be regarded as an artificial object of mathematics because there is no natural crystal prototype as there is for the other Platonic solids. See Benno Artmann, “Antike Darstellungen des Ikosaeders”, in: Rainer Schulze-Pillot (Ed.), *Mitteilungen der Deutschen Mathematiker Vereinigung*, Bd. 13, Heft 1/2005, p.46-50, here p.46.

⁷³ The German mathematician Felix Klein expresses himself in his (posthumously published) lecture about the history of mathematics with the following: “of particular interest, it could be that Monge skipped directly from simple drawings to modeling” („Von besonderem Interesse dürfte es vielleicht sein, dass Monge von dem bloßen Zeichnen bereits zum Modellieren übergeht.“) See Felix Klein, *Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert, Teil I*, Berlin 1926, p.78.

⁷⁴ In 1868, the Italian mathematician Beltrami named the first model for a surface of constant negative curvature: the pseudosphere, which is a surface of rotation created by using the tractrix. In 1871, Felix Klein spoke out for the production and usage of such pictures in order to better demonstrate the abstract speculations of non-Euclidian geometry (“sehr abstrakten Spekulationen”). See Klaus Mainzer, *Geschichte der Geometrie*, Mannheim, Wien, Zürich 1980, p.170.

⁷⁵ Fermat declares in his *Isagoge* that the equation can be easily made concrete („Die Gleichungen kann man aber bequem versinnlichen“) by using an adequate coordinate system. See Pierre de Fermat, “Einführung in die ebenen und körperlichen Örter (Ad locis planis et solidis isagoge)”, in Heinrich Wieleitner (Ed.), *Ostwalds Klassiker der exakten Wissenschaften, Nr.208*, Leipzig 1923, p.7 On development of the coordinate method and analytical geometry see Peter Schreiber, Christoph J. Scriba, *5000 Jahre Geometrie. Geschichte, Kulturen, Menschen*, first corrected reprint, Berlin et al. 2002, p.300ff.

ways.⁷⁶ This creates a paradoxical situation in connection with the epistemic function of images: one is familiar with images, recognizes the legitimating power and even develops new visualization methods, yet simultaneously distrusts the images and denies their argumentative power.

Vollmer refers to the handmade models for teaching and research purposes and produced these in new materials, thus bridging Snow's so-called two cultures. As a side note: Vollmer's progression in the history of mathematics concerning the choice of her themes – from the Euclidian sphere to the models of non-Euclidian geometry of the 19th century – is paralleled by changes in her materials – starting with wood and bronze and moving into plastic and foils printed with grids – all becoming more artificial. A free interpretation, like for example Naum Gabos *Translucent Variation of a Spheric Theme*,⁷⁷ does not occur, but Vollmer nevertheless distances herself from the intention to create sculptures for the purpose of illustrating mathematics: "I can't say I do the mathematical form because of mathematics."⁷⁸

For Ruth Vollmer the mathematical images offer objectivity and this is the cause of their beauty: "the mathematical form is so beautiful because it's so objective! It is not beautiful because it is made by a person, or looked at by a person. It is outside of the particular."⁷⁹ This is also attested to by the concept artist Mel Bochner in an interview: "Ruth, on the other hand, did believe in the veracity of scientific images,"⁸⁰ and ascribes to her a romantic view and belief in the revelation of science through the structures of nature. Vollmer displays a Platonic belief when she speaks of the forms she invented: "I suppose they have existed in Mathematics before, but they have not been manifested visibly."⁸¹ Bochner also sees the scientific images bound to specific cultural and temporal systems of representation.⁸²

Today one could speak of the contingency of the images and that this applies in a double sense to the two models received by Vollmer: concepts of analytical geometry, such as the choice of the

⁷⁶ See Heinz Otto Peitgen, "Bedeutungswandel der Bilder in der Medizin – Bilder zwischen Deutung und Wissen", in: Barbara Könches, Peter Weibel (Ed.), *unSICHTBARes. Algorithmen als Schnittstellen zwischen Kunst und Wissenschaft*, Bern 2005, p.98-121, here p.98.

⁷⁷ Anna Vallye compares the work *Pseudosphere* of Ruth Vollmer with Naum Gabo's *Translucent Variation of a Spheric Theme* (1951) and points out Vollmer's proximity to Minimal Art in her avoidance of personal intervention and interpretation to a large extent. When recasting the models into other materials, she also alludes to the ready-made strategy of Marcel Duchamp. See Vallye, "The Reenchantment of the world: Ruth Vollmer's Science", l.c., p.100f.

⁷⁸ See Larsen, "An Interview with Ruth Vollmer", l.c., p.210.

⁷⁹ See *ibid.*

⁸⁰ See Nadja Rottner, "In Conversation with Mel Bochner", in: Nadja Rottner, Peter Weibel (Ed.), *Ruth Vollmer 1961-1978, Thinking the Line*, Stuttgart 2006, p.212-213, here p. 213.

⁸¹ The question about the ontological status of mathematical objects leads to different positions in the philosophy of mathematics, one such position being Platonism. In the Platonic view, mathematical objects exist independently from human beings and human awareness, but it is possible, as Plato shows in his *Menon* dialogue, to discover them (Anamnesis). That means mathematical objects are discoveries and not human inventions. See Bettina Heintz, *Die Innenwelt der Mathematik. Zur Kultur und Praxis einer beweisenden Disziplin*, Wien, New York 2000, p.39. For Vollmer's quote see Ruth Vollmer, "Statement 1968", in Nadja Rottner, Peter Weibel (Ed.), *Ruth Vollmer 1961-1978, Thinking the Line*, Stuttgart 2006, p.203.

⁸² See Nadja Rottner, "In Conversation with Mel Bochner", l.c., p.213.

coordinate system, have some influence on the final appearance and the decision for or against the production is determined by the necessity in the visual representation of an existing mathematical problem. Furthermore, there are an immense number of parameters that are dependent on the technical and artistic abilities of the producer.

Mel Bochner addresses the phenomenon of changes of the medial representation in his work, especially those of pictorial and non-pictorial representations, the conditions of the visualization and the corresponding visualization methods. The discrepancy between the ideally imagined world of mathematics and its application in the world serves as a source of inspiration.

Bochner arrived in New York in 1964 after having studied at the Carnegie Institute of Technology in Pittsburgh. He worked as studio assistant to Ruth Vollmer in 1965, and in 1966 created the first exhibition of concept art, *Working Drawings and Other Visible Things on Paper Not Necessarily Meant To Be Viewed As Art*, in which he emerged both as artist and curator when he presented four identical books of photocopies of artwork from artist friends and scientific diagrams.

In his photo series of the late 1960s, he brilliantly investigated the conditions of central perspective, one of the most important mathematically based methods for the creation of an illusory pictorial space, which – as demonstrated earlier by Erwin Panofsky – is a symbolic form (“symbolische Form”) and is subject to cultural premises.⁸³

His presentation at first of abstract grids in one-point perspective can be seen as an allusion to Alberti’s “pavimento”, which can be found in many Renaissance paintings. In his treatise *Della Pittura*, Alberti provided the painters of his time with mathematically based guidelines for their craft, with the intention of utilizing Brunelleschi’s findings for a revitalized and truthful art.⁸⁴

For the transferring of the subject onto the picture plane, Alberti also presents a process of using a grid of threads in order to divide the picture into discrete parts, anticipating what would occur much later in digital pictures.⁸⁵ Thus, the postcard for LeWitt’s Munich exhibition can be regarded as a reference to Alberti’s window-view (“perspicere”).

Bochner deconstructed this process by using the camera as a perspective device to unmask perspective as an idealized construction of space. Similarly, he inserts “cutouts” in his drawings, like

⁸³ See Erwin Panofsky, “Perspektive als symbolische Form”, in: Erwin Panofsky, *Aufsätze zu Grundfragen der Kunstwissenschaft*, Berlin 1974, p.99-167.

⁸⁴ The Florentine master builder Filippo Brunelleschi is regarded as the person who rediscovered the central perspective around 1425, which was facilitated by the appearance of Ptolemaios’ *Geographia*, as Samuel Edgerton shows. See Samuel Edgerton, *Die Entdeckung der Perspektive*, München 2002, p.85ff. The treatise *De Pictura* (*About Painting*) was published in Latin and later in Old Italian (*Della Pittura*). See Leon Battista Alberti, *Über die Malkunst*, Oskar Bätschmann, Sandra Gianfreda (Ed.), second unchanged edition, Darmstadt 2007.

⁸⁵ On the prehistory of digital pictures see Michael Rottmann, *Das digitale Bild als Visualisierungsstrategie der Mathematik*, in: Ingeborg Reichle, Steffen Siegel, Achim Spelten (Ed.), *Verwandte Bilder. Die Fragen der Bildwissenschaft*, Berlin 2007, p.281-296.

Untitled (Holes) (1967) in order to disrupt the perspective system with empty spaces and thereby destroying the illusion. The “cutouts” open up the view of the depicted object, a flat aperture or “cutout” of the gridded paper, which also serves as the base of the image. In the drawing *Untitled (Perspective Overlay)*, he also juxtaposes different perspective systems (one-point, two-point, isometric and orthogonal grids) and brings the situation to a head by overlaying them so that the difference between reality and its construction is evident.

Bochner intensifies his studies in paper works, such as *Untitled (Deformations)* (1967) and shows the mutation of grids through distortions. He folded the drawing paper, crumpled it, wrapped it around corners or cylinders. The geometry on the surface itself does not change, but instead the appearance of the grids on the paper does when viewed from outside. This approach shows his deep interest in the processes as they relate to the different placements of objects in space, in mapping processes of cartography, which he describes as highly abstract systems in his article *The Serial Attitude*.

In his *Measurements*, which he started in 1969, Bochner measures exhibition spaces and shows the referential conditions of numbers. The pure number, as a self-referential entity, cannot describe the room with its physical and phenomenological properties. A frame of reference must be based on a one-dimensional measurement (length, width or height), such as those used by LeWitt. The coding of the perception of space is a utopia, since with every change in the media, certain properties of the media are lost. Thus mathematical model and formula are not identical, since the mathematical language is, according to Sybille Krämer, an “operational script” in contrast to the image.⁸⁶ A property may be lost in the corresponding picture but it is compensated for through the acquisition criticality value, for example when structures can be recognized.

The language of mathematics does not work in a deictic way; instead it deals with the conditions of the “proving discipline” and does not refer beyond that. The coupling of the mathematical language and its symbols with pictures and objects of empirical reality is precisely what is necessary for the modeling of this reality and is also what makes this modeling feasible. Numbers are no longer used solely to construct space but also to represent it.

From the vantage point concerning the change of media, one can trace a shifting relationship between the object and its representation or different forms of its representation, such as in the works of Vollmer and Bochner. It can also be seen in LeWitt’s oeuvre – starting with the *Structures* and progressing into *Walldrawings*. This also occurs in his *Lines to Specific Points*, as he unifies two systems of symbols within one sheet. In a square with “marked” points, which serves as the framework, he uses

⁸⁶ In contrast to phonetic script, the operative script of mathematical calculus allows an operating (in a symbolic way) with the objects of mathematics. See Sybille Krämer, “Kalküle als Repräsentation. Zur Genese des operativen Symbolismus in der Neuzeit”, in: Hans-Jörg Rheinberger, Michael Hagner, Bettina Wahrig-Schmidt (Ed.), *Räume des Wissens. Repräsentation, Codierung, Spur*, Berlin 1997, p.111-122, here p.111.

relative information to construct a growing picture, which is composed of auxiliary and signifying lines and is distinguished through varying intensities of color. In contrast to the “coordinate method” of the Baroque, he interweaves the construction text and the diagram, as was found in classical geometric constructions, for example, in Euclid’s *Elements*.

Hanne Darboven, a friend of LeWitt, also works in a similar way in her *Kleinen Konstruktionen*, when she uses numbers in her axis systems that are frequently based on squares and when she designs systematic rows in a unique notation and in her own arithmetic.

Bochner also operates within a square framework in his sheets *Counting Alternatives*, *The Wittgenstein Illustrations*. He presents varying handwritten, linear reckonings, which fork and branch within a system comprised of a square (symbol of rationalism) and its subdivisions. This is less of an illustration of Wittgenstein’s work that accompanies the piece in an attached sheet and more a doubting of rational thinking, which he displays in his pictorial compositions.

In his group of works *Theory of Sculpture*, he fixates on an early strategy of visualization developed by the school of Pythagoras. The Pythagorean school in the 6th century probably used stones of equal size and shape (Psäphoi) which they placed in patterns to visualize various arithmetic subjects, thereby discovering relationships between numbers and arriving at basic arithmetic statements.⁸⁷ Early evidence of this is found in Aristotle’s *Metaphysics*, in which he reports that Eurytos, a follower of Pythagoras, reproduced the so-called figurized numbers in the shape of a triangle or square.

When Bochner displays the Cantor set, a fractal in mathematics, in his drawing *Cantor’s Paradox*, he suggests the limits of pictorial representation of formal subjects. The Cantor set can be exactly formally described, but eludes visual representation, because a certain level of representation falls short of the human limit of resolution.⁸⁸

In this sense it is no longer about making the invisible visible, as Paul Klee formulates it in an aperçu, but instead about the visualization of previously non-pictorial subjects. This is the theme and challenge of the “new” art, as Bochner says, “Old Art attempted to make the non-visible (energy, feelings) visual (marks). The New Art is attempting to make the non-visual (mathematics) visible (concrete).”⁸⁹

The concepts of the “exact” science of mathematics, which change the thoughts and the approach of humans to the world, become realized in the art of the New York Circle artists, but “different” because

⁸⁷ See Oskar Becker, *Grundlagen der Mathematik in geschichtlicher Entwicklung*, Frankfurt/M. 1975, p.34ff.

⁸⁸ Richard S. Field sees a connection to the new pedagogy of “New Math”, which is based on set theory, and therefore sees Bochner’s work as an homage to the father of set theory Georg Cantor. See Richard S. Field, “Mel Bochner: Thought Made Visible”, in: Lesley K. Baier (Ed.), *Mel Bochner: Thought Made Visible 1966-1973*, New Haven 1995, p.15-74, here p.21.

⁸⁹ See Mel Bochner, “Primary Structures”, in: *Arts Magazine*, Vol. 40, No. 8 (1966), p.32-35, here p.35, quoted after Rottner, “In Conversation with Mel Bochner”, l.c., p.213.

in system art it is not about work on the precise language of mathematics, but instead about the questioning of the basic conditions of art, the realization of declared ideas and the gaining of new insights through aesthetic experiences.